

# Related-Key Kasumi attacks

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Ján Kotrady

# Obsah

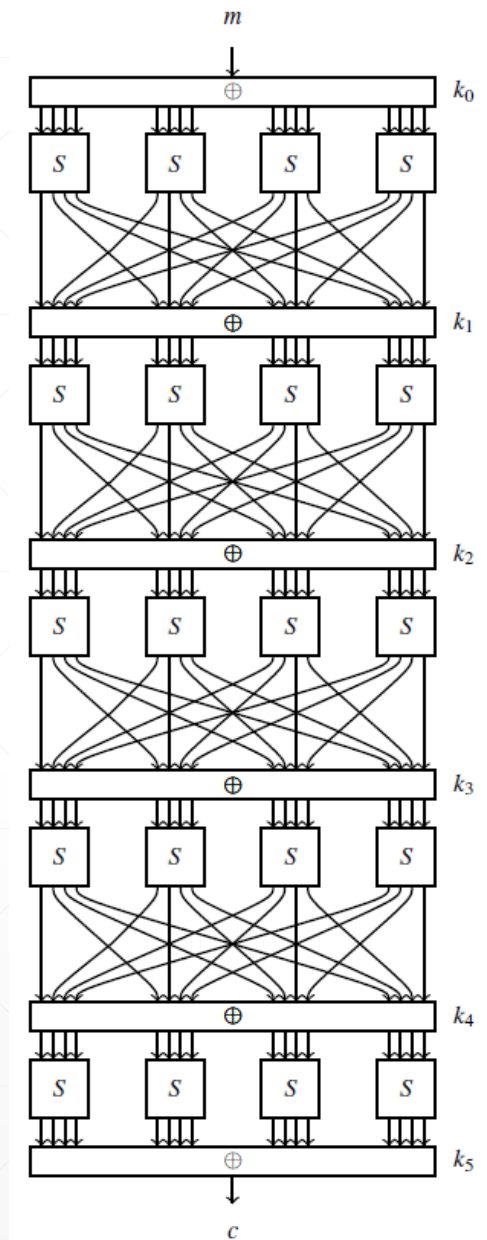
- Diferenčná kryptoanalýza
  - Boomerang attack\*
  - Amplified Boomerang Attack\*
  - Related-Key Attack\*
  - Related-Key Boomerang Attack\*
  - Related-Key Rectangle Attack\*
  - KASUMI Related-Key Rectangle Attack
  - KASUMI Related-Key Sandwich (Boomerang) Attack\*
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# Diferenčná kryptoanalýza „myšlienka“

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# Blokové šifry

- Najúčinnnejšia kryptoanalýza ( $\neg$ DES)
- Základ: „rundy“, Feistelová sieť
- **CPA – Chosen plain-text attack**
- Ciele:
  - „Zbaviť sa kľúča“
  - Zistiť kľúč
  - Útok hrubou silou jednoduchší



# XOR – základný princíp

- $\oplus$
- XOR šifra:  $k_1$  kľúč,  $e_{k_1}(m) = m \oplus k_1$ ,  $d_{k_1}(m) = m \oplus k_1$
- $d_{k_1}(e_{k_1}(m)) = m \oplus k_1 \oplus k_1$

XOR operácie			
1	$\oplus$	1	0
1	$\oplus$	0	1
0	$\oplus$	1	1
0	$\oplus$	0	0

$k_1=101101$			
1	$\oplus$	1	0
0	$\oplus$	0	0
1	$\oplus$	1	0
1	$\oplus$	1	0
0	$\oplus$	0	0
1	$\oplus$	1	0

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- $(m_1 \oplus k_1) \oplus (m_2 \oplus k_2)$

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**Práve sme pochopili  
diferenčnú kryptoanalýzu**

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diferenčnú kryptoanalýzu  
nasleduje jej aplikácia 😞**

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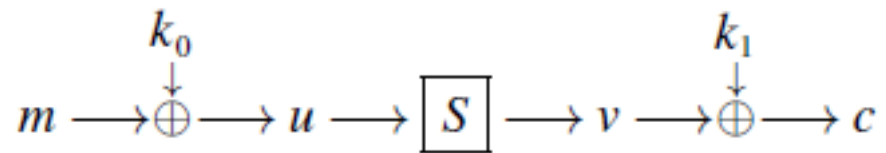
# CipherOne

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# CipherOne

$$c = S[m \oplus k_0] \oplus k_1$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b



$\text{CIPHERONE}(m_0, k_0    k_1)$	$\text{CIPHERONE}(m_1, k_0    k_1)$
$u_0 = m_0 \oplus k_0$	$u_1 = m_1 \oplus k_0$
$v_0 = S[u_0]$	$v_1 = S[u_1]$
$c_0 = v_0 \oplus k_1$	$c_1 = v_1 \oplus k_1$

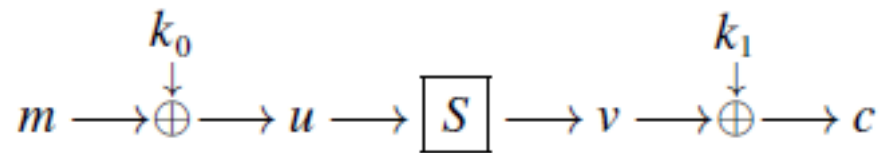
## Aplikácia v praxi

- $m_0 \oplus m_1 = (m_0 \oplus k_0) \oplus (m_1 \oplus k_0) = u_0 \oplus u_1$

# CipherOne

$$c = S[m \oplus k_0] \oplus k_1$$

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CIPHERONE( $m_0, k_0    k_1$ )	CIPHERONE( $m_1, k_0    k_1$ )
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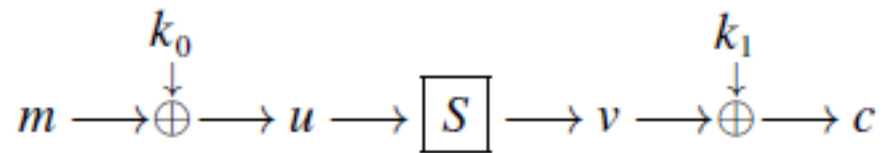
## Aplikácia v praxi

- $m_0 \oplus m_1 = (m_0 \oplus k_0) \oplus (m_1 \oplus k_0) = u_0 \oplus u_1$
- $(m_0, c_0), (m_1, c_1)$  - pár

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## Aplikácia v praxi

- $m_0 \oplus m_1 = (m_0 \oplus k_0) \oplus (m_1 \oplus k_0) = u_0 \oplus u_1$
- $(m_0, c_0), (m_1, c_1)$  - pár
- $u_0 \oplus u_1 = S^{-1}[t \oplus c_0] \oplus S^{-1}[t \oplus c_1], \forall t$

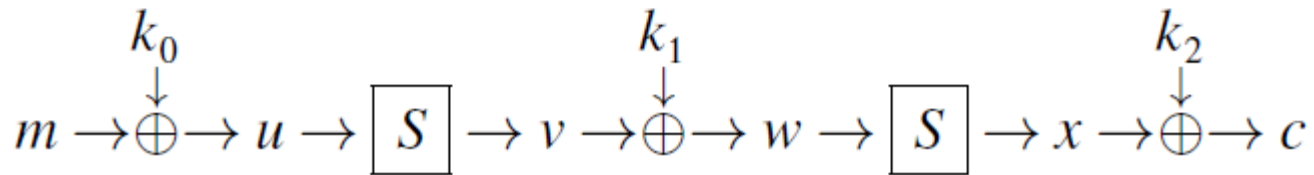
# CipherTwo

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# CipherTwo

$$c = S[S[m \oplus k_0] \oplus k_1] \oplus k_2$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b



CIPHERTWO( $m_0, k_0    k_1    k_2$ )	CIPHERTWO( $m_1, k_0    k_1    k_2$ )
$u_0 = m_0 \oplus k_0$	$u_1 = m_1 \oplus k_0$
$v_0 = S[u_0]$	$v_1 = S[u_1]$
$w_0 = v_0 \oplus k_1$	$w_1 = v_1 \oplus k_1$
$x_0 = S[w_0]$	$x_1 = S[w_1]$
$c_0 = x_0 \oplus k_2$	$c_1 = x_1 \oplus k_2$

## Aplikácia v praxi

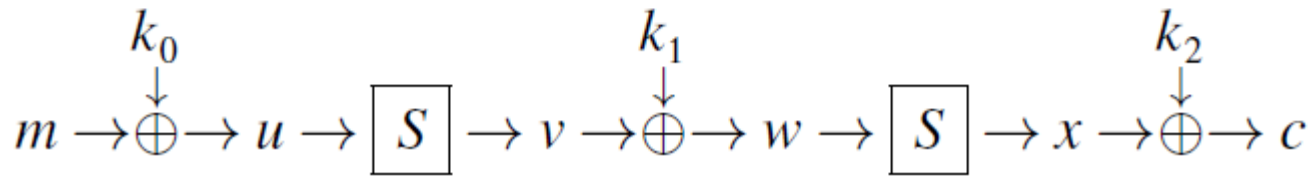
- $m_0 \oplus m_1 = (m_0 \oplus k_0) \oplus (m_1 \oplus k_0) = u_0 \oplus u_1$
- $(m_0, c_0), (m_1, c_1)$



# CipherTwo

$$c = S[S[m \oplus k_0] \oplus k_1] \oplus k_2$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b



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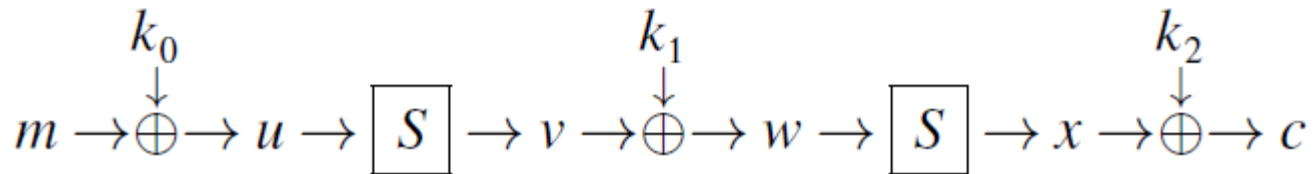
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- $w_0 \oplus w_1 = v_0 \oplus v_1 = S^{-1}[t \oplus c_0] \oplus S^{-1}[t \oplus c_1], \forall t$

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CIPHERTWO( $m_0, k_0    k_1    k_2$ )	CIPHERTWO( $m_1, k_0    k_1    k_2$ )
$u_0 = m_0 \oplus k_0$	$u_1 = m_1 \oplus k_0$
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- Nevieme overiť tip  $t$

Inputs and output relations for  $i$  and  $j = i \oplus \mathbf{f}$  across  $S[\cdot]$ .

$i$	$j$	$S[i]$	$S[j]$	$S[i] \oplus S[j]$
0	f	6	b	d
1	e	4	9	d
2	d	c	a	6
3	c	5	8	d
4	b	0	d	d
5	a	7	3	4
6	9	2	f	d
7	8	e	1	f
8	7	1	e	f
9	6	f	2	d
a	5	3	7	4
b	4	d	0	d
c	3	8	5	d
d	2	a	c	6
e	1	9	4	d
f	0	b	6	d

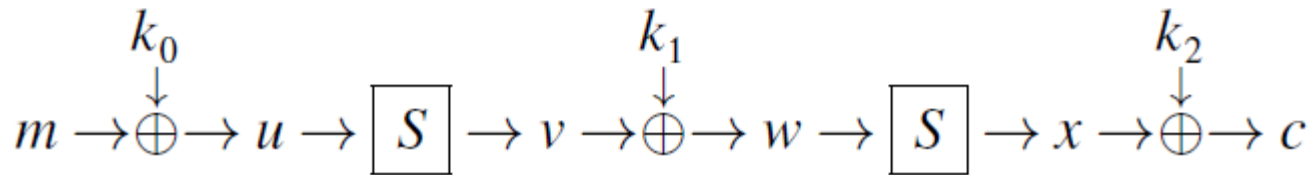
## Analýza S-boxu $S[x]=f$

- $j \oplus i = \mathbf{f}, j = i \oplus \mathbf{f}$
- $\mathbf{d} = 10/16$

# CipherTwo

$$c = S[S[m \oplus k_0] \oplus k_1] \oplus k_2$$

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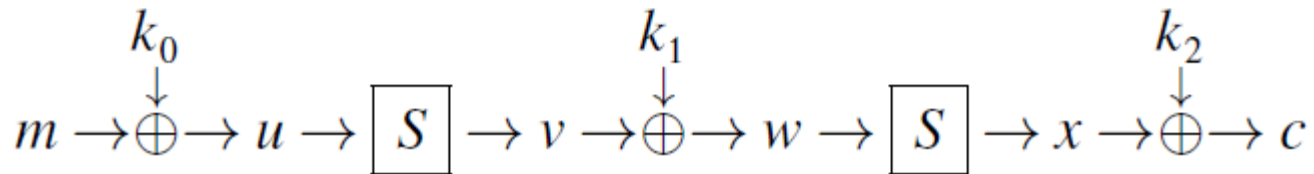
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$$c = S[S[m \oplus k_0] \oplus k_1] \oplus k_2$$

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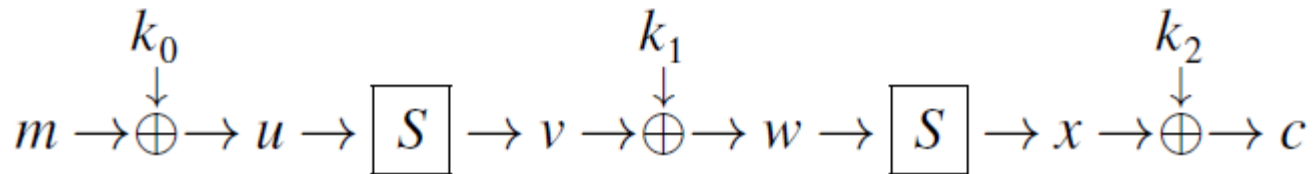
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- Nevieme overiť tip  $t$
- $m_0 = f \oplus m_1$

# CipherTwo

$$c = S[S[m \oplus k_0] \oplus k_1] \oplus k_2$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
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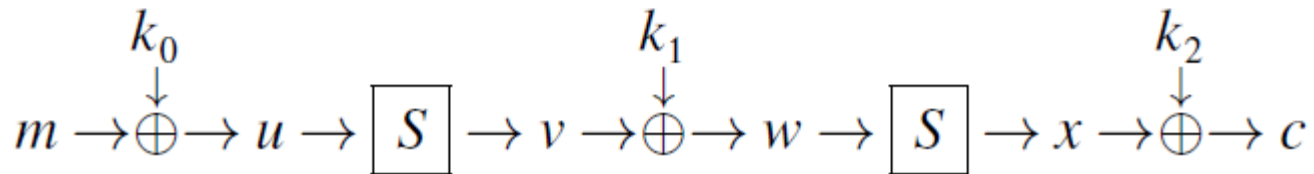
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- Nevieme overiť tip  $t$
- $m_0 = f \oplus m_1$
- $P\{S[u_0] \oplus S[u_1] = \mathbf{d}\} = \frac{10}{16}$

# CipherTwo

$$c = S[S[m \oplus k_0] \oplus k_1] \oplus k_2$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
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- $m_0 = f \oplus m_1$
- $P\{S[u_0] \oplus S[u_1] = \mathbf{d}\} = \frac{10}{16}$
- Counter  $T_1, \dots, T_{|k|}$

# CipherThree

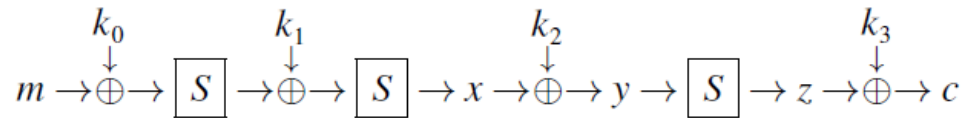
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# CipherThree

$$c = S[S[S[m \oplus k_0] \oplus k_1] \oplus k_2] \oplus k_3$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b



## $\Delta_{out}$

$\Delta_{in}$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

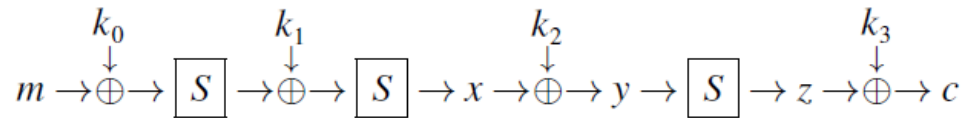
## Diferenčná tabuľka S-box charakteristika

- $(\alpha, \beta), \alpha \rightarrow \beta$

# CipherThree

$$c = S[S[S[m \oplus k_0] \oplus k_1] \oplus k_2] \oplus k_3$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b



## $\Delta_{out}$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	2	2	-	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

$\Delta_{in}$

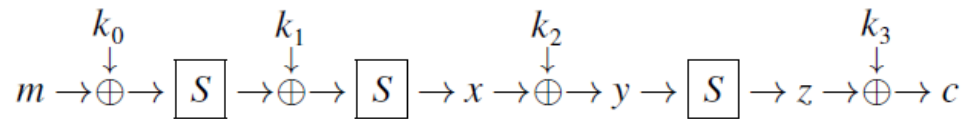
## Diferenčná tabuľka S-box charakteristika

- $(\alpha, \beta), \alpha \rightarrow \beta$
- $P_1\{S[u_0] \oplus S[u_1] = d\} = \frac{10}{16}$

# CipherThree

$$c = S[S[S[m \oplus k_0] \oplus k_1] \oplus k_2] \oplus k_3$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b



## $\Delta_{out}$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

$\Delta_{in}$

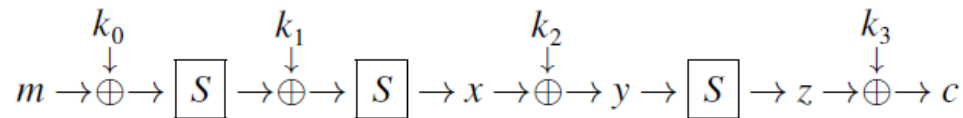
## Diferenčná tabuľka S-box charakteristika

- $(\alpha, \beta), \alpha \rightarrow \beta$
- $P_1\{S[u_0] \oplus S[u_1] = \mathbf{d}\} = \frac{10}{16}$
- $P_2\{S[\mathbf{d}] = \mathbf{c}\} = \frac{6}{16}$

# CipherThree

$$c = S[S[S[m \oplus k_0] \oplus k_1] \oplus k_2] \oplus k_3$$

$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b



## $\Delta_{out}$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	2	2	-	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

$\Delta_{in}$

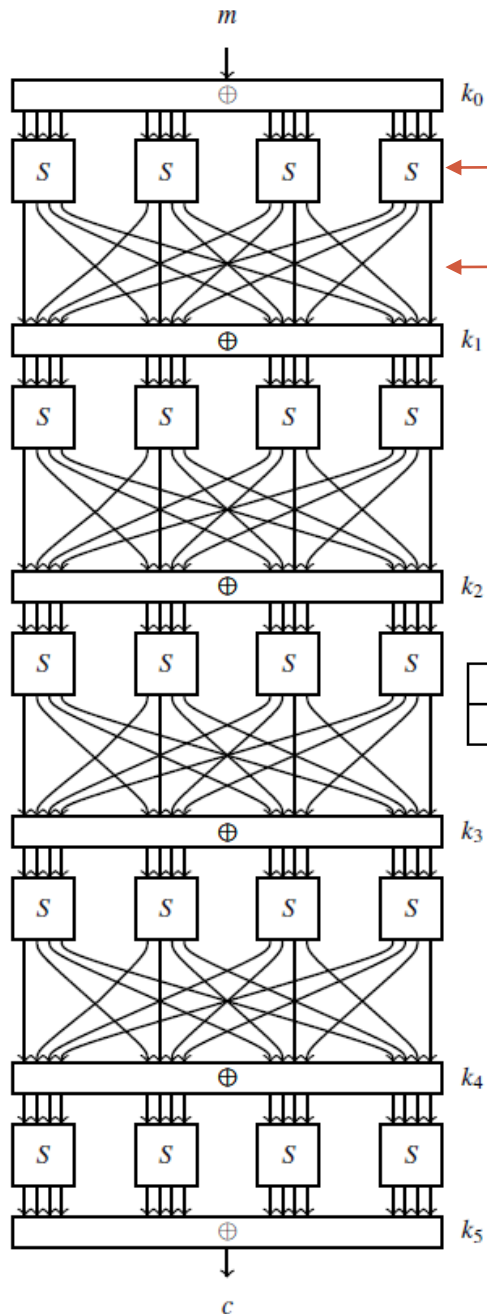
## Diferenčná tabuľka S-box charakteristika

- $(\alpha, \beta), \alpha \rightarrow \beta$
- $P_1\{S[u_0] \oplus S[u_1] = d\} = \frac{10}{16}$
- $P_2\{S[d] = c\} = \frac{6}{16}$
- $P_1 \& P_2 = \frac{10}{16} \frac{6}{16}$

# CipherFour

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# CipherFour



S-box

P-box

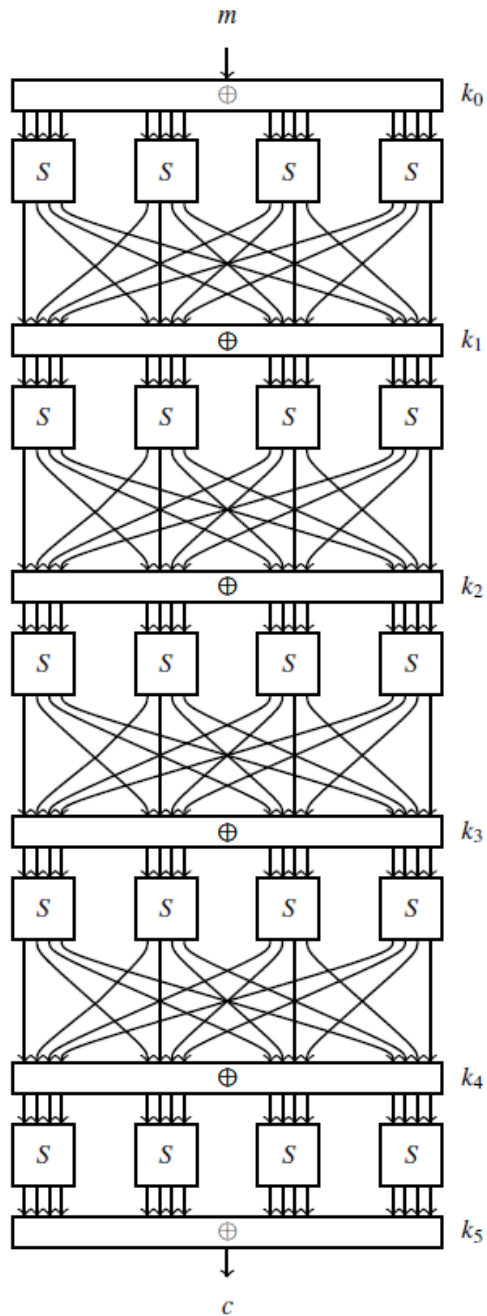
$x$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S[x]$	6	4	c	5	0	7	2	e	1	f	3	d	8	a	9	b

$i$	0	1	2	3	4	5	6	7
$P[i]$	0	4	8	12	1	5	9	13
$i$	8	9	10	11	12	13	14	15
$P[i]$	2	6	10	14	3	7	11	15

## Spájanie charakteristík

- S-box – nelineárne
- P-box – permutácia

# CipherFour



$\Delta_{out}$

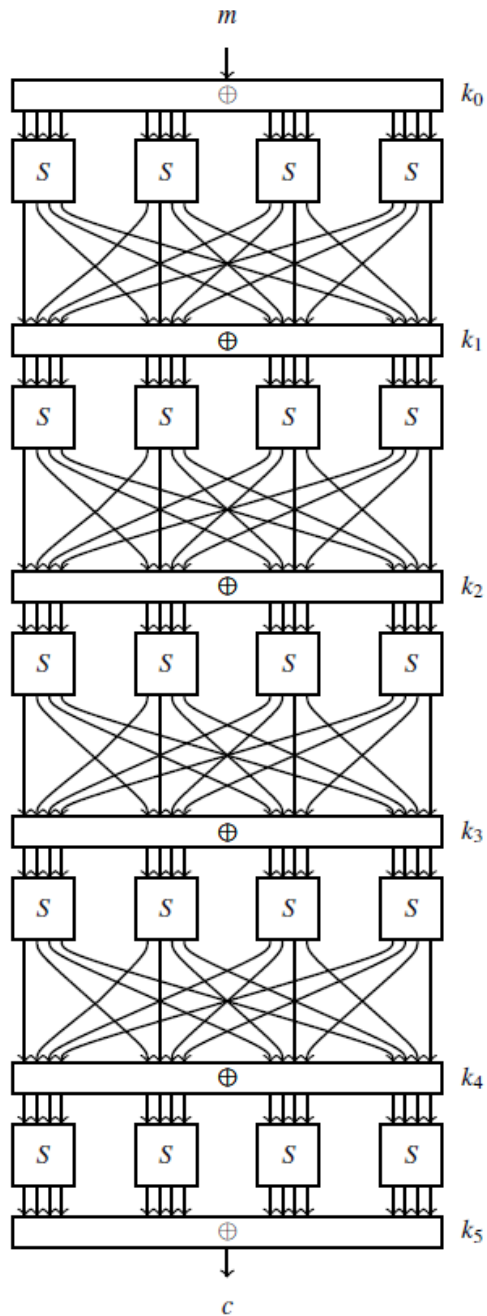
	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

$\Delta_{in}$

## Spájanie charakteristík

- $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \xrightarrow{S} (\beta_1, \beta_2, \beta_3, \beta_4)$
- $(\beta_1, \beta_2, \beta_3, \beta_4) \xrightarrow{P} (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$
- $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \xrightarrow{R} (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$

# CipherFour



$\Delta_{out}$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

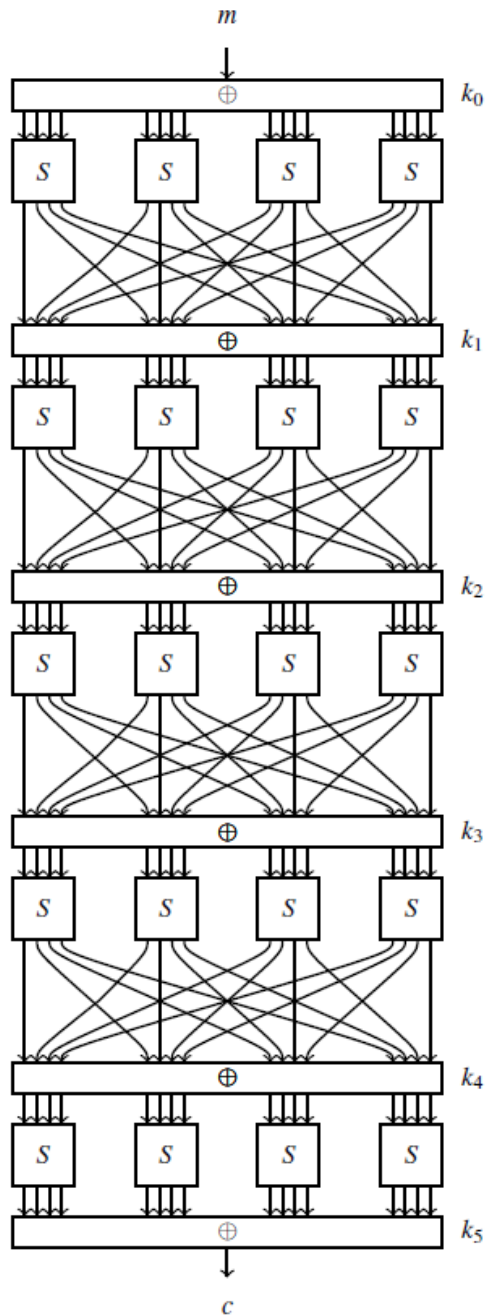
$\Delta_{in}$

## Spájanie charakteristík

- $(0,0,0,f) \xrightarrow{S} (0,0,0,d)$
- $(0,0,0,d) \xrightarrow{P} (1,1,0,1)$
- $P_1 = \frac{10}{16}$



# CipherFour



## $\Delta_{out}$

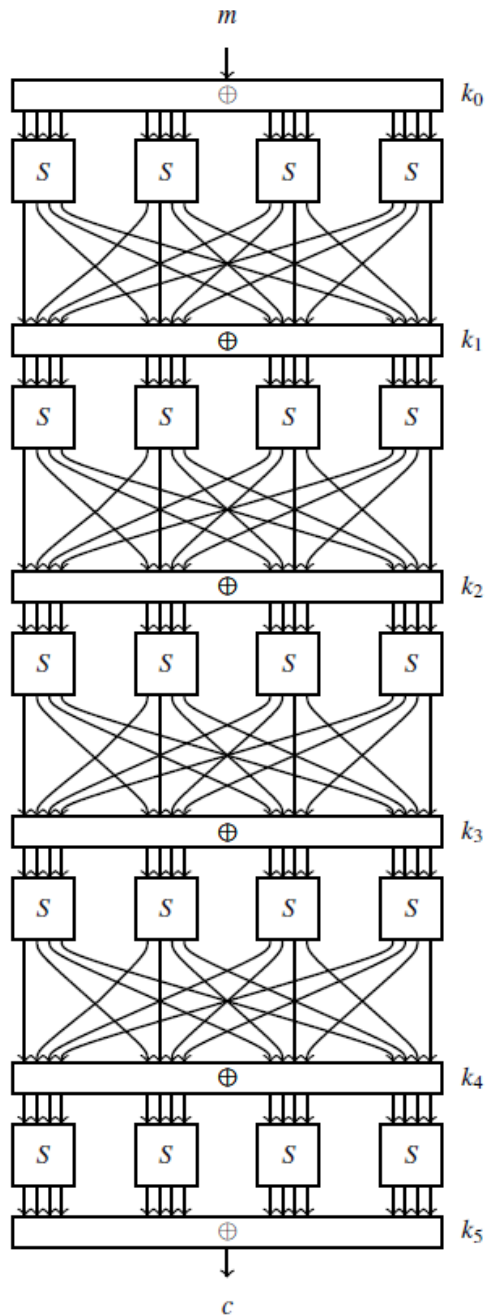
	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

## $\Delta_{in}$

## Spájanie charakteristík

- $(0,0,0,f) \xrightarrow{S} (0,0,0,d)$
- $(0,0,0,d) \xrightarrow{P} (1,1,0,1)$
- $P_1 = \frac{10}{16}$
- $(1,1,0,1) \xrightarrow{S} (2,2,0,2)$
- $(2,2,0,2) \xrightarrow{P} (0,0,d,0)$
- $P_2 = \left(\frac{6}{16}\right)^3$
- $P_1 \& P_2 = \frac{10}{16} \left(\frac{6}{16}\right)^3 = \frac{135}{4096}$

# CipherFour



$\Delta_{out}$

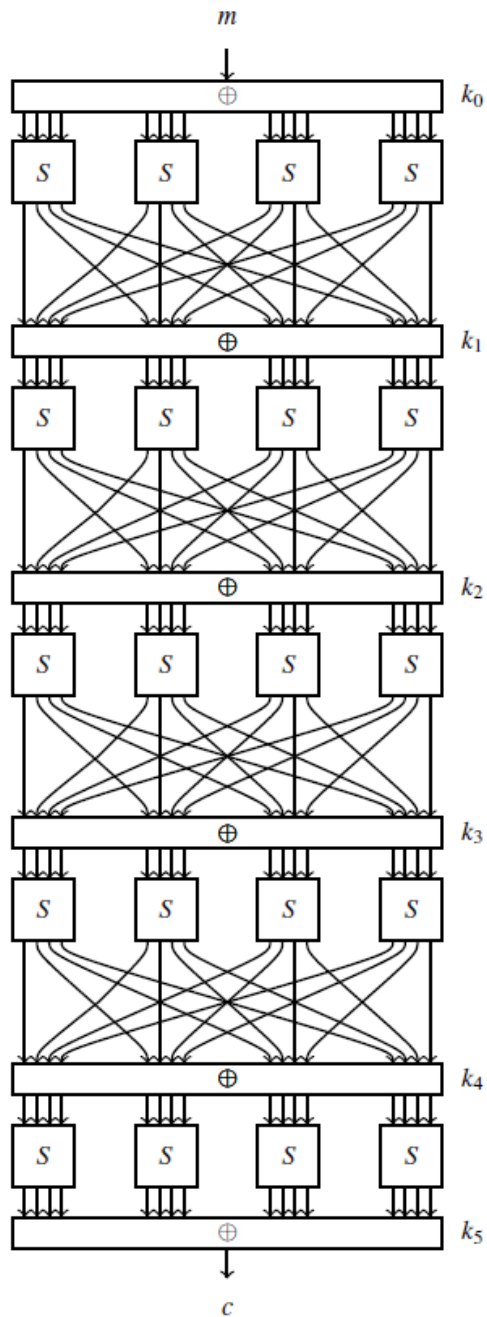
	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

$\Delta_{in}$

## Spájanie charakteristík

- $(0,0,2,0) \xrightarrow{S} (0,0,2,0)$
- $(0,0,2,0) \xrightarrow{P} (0,0,2,0)$
- $P_1 = \frac{6}{16}$
- $(0,0,2,0) \xrightarrow{S} (0,0,2,0)$
- $(0,0,2,0) \xrightarrow{P} (0,0,2,0)$
- $P_2 = \frac{6}{16}$
- $P_1 \ \& \ P_2 = \left(\frac{6}{16}\right)^2$

# CipherFour



$\Delta_{out}$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

$\Delta_{in}$

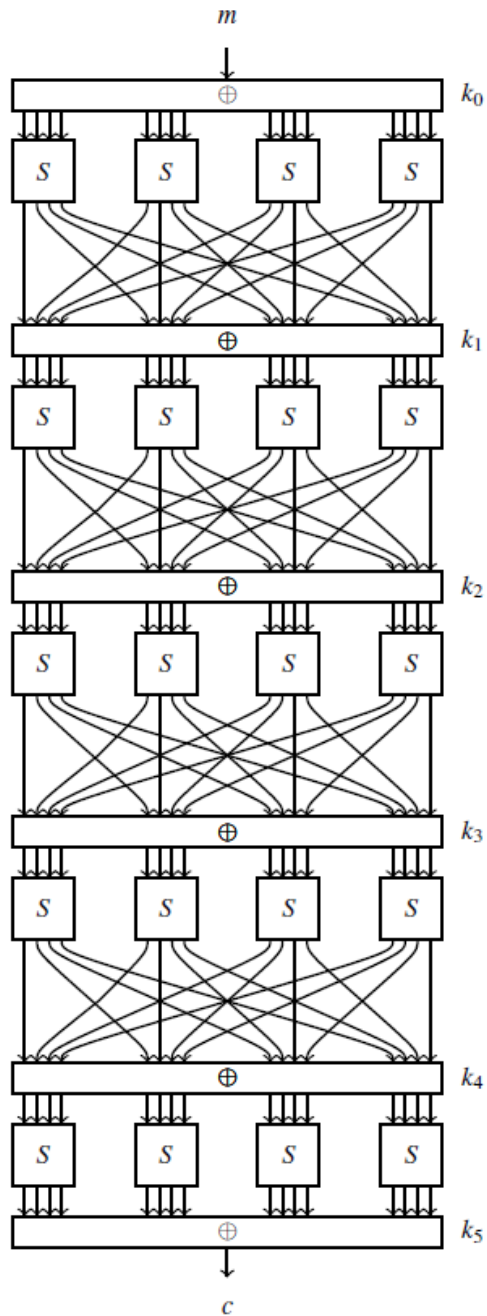
## Spájanie charakteristík

- $(0,0,2,0) \xrightarrow{R} (0,0,2,0)$
- $(0,0,2,0) \xrightarrow{R} (0,0,2,0)$
- $(0,0,2,0) \xrightarrow{R} (0,0,2,0)$
- $(0,0,2,0) \xrightarrow{R} (0,0,2,0)$
- $P = \left(\frac{6}{16}\right)^4 = \frac{135}{4096}$

# Zavedenie diferencií

- $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \xrightarrow{S} (\beta_1, \beta_2, \beta_3, \beta_4)$
  - $(\beta_1, \beta_2, \beta_3, \beta_4) \xrightarrow{P} (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$
  - $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \xrightarrow{R} (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$
  - $(0, 0, 2, 0) \xrightarrow{R} (0, 0, 2, 0) \xrightarrow{R} (0, 0, 2, 0) \xrightarrow{R} (0, 0, 2, 0) \xrightarrow{R} (0, 0, 2, 0)$
  - $(0, 0, 2, 0) \xrightarrow{R} ? \xrightarrow{R} ? \xrightarrow{R} ? \xrightarrow{R} (0, 0, 2, 0)$
  - $(0, 0, 2, 0) \xrightarrow{R} (0, 0, 0, 2) \xrightarrow{R} (0, 0, 0, 1) \xrightarrow{R} (0, 0, 1, 0) \xrightarrow{R} (0, 0, 2, 0)$
  - $(0, 0, 2, 0) \xrightarrow{R} (0, 0, 0, 2) \xrightarrow{R} (0, 0, 1, 0) \xrightarrow{R} (0, 0, 2, 0) \xrightarrow{R} (0, 0, 2, 0)$
  - $(0, 0, 2, 0) \xrightarrow{R} (0, 0, 2, 0) \xrightarrow{R} (0, 0, 0, 2) \xrightarrow{R} (0, 0, 1, 0) \xrightarrow{R} (0, 0, 2, 0)$
-

# CipherFour



## $\Delta_{out}$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	6	-	-	-	-	2	-	2	-	-	2	-	4	-
2	-	6	6	-	-	-	-	-	-	2	2	-	-	-	-	-
3	-	-	-	6	-	2	-	-	2	-	-	-	4	-	2	-
4	-	-	-	2	-	2	4	-	-	2	2	2	-	-	2	-
5	-	2	2	-	4	-	-	4	2	-	-	2	-	-	-	-
6	-	-	2	-	4	-	-	2	2	-	2	2	2	-	-	-
7	-	-	-	-	-	4	4	-	2	2	2	2	-	-	-	-
8	-	-	-	-	-	2	-	2	4	-	-	4	-	2	-	2
9	-	2	-	-	-	2	2	2	-	4	2	-	-	-	-	2
a	-	-	-	-	2	2	-	-	-	4	4	-	2	2	-	-
b	-	-	-	2	2	-	2	2	2	-	-	4	-	-	2	-
c	-	4	-	2	-	2	-	-	2	-	-	-	-	-	6	-
d	-	-	-	-	-	-	2	2	-	-	-	-	6	2	-	4
e	-	2	-	4	2	-	-	-	-	-	2	-	-	-	-	6
f	-	-	-	-	2	-	2	-	-	-	-	-	-	10	-	2

## $\Delta_{in}$

## Filtrovanie

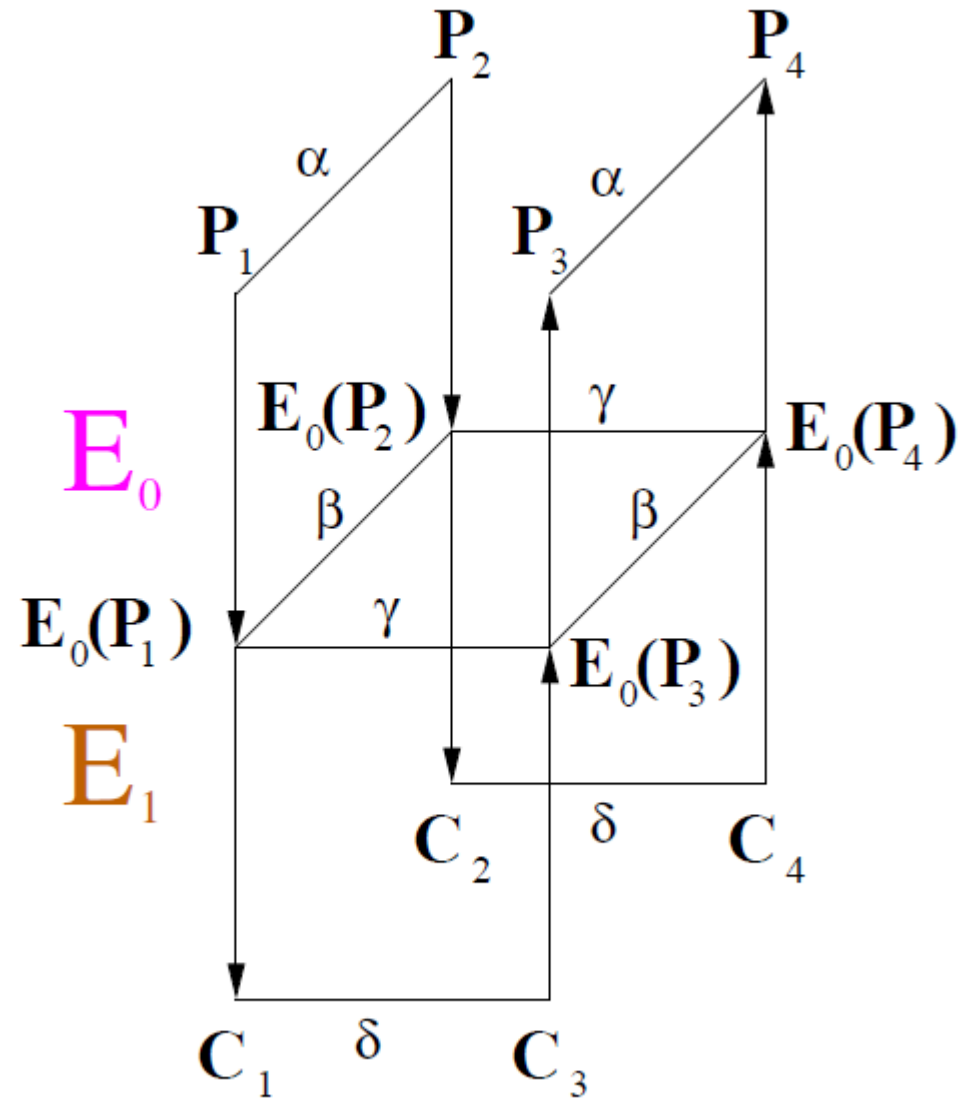
- $(0,0,2,0) \xrightarrow{S} (0,0,h,0)$
- $h \in \{1,2,9,a\}$
- $\frac{5310}{65536} \approx 0.08$  - diferencia
- $\frac{7387}{65536} \approx 0.11$  - filtrovanie
- $t \times 0.08$  - diferencia
- $t \times 0.11$  - filter
- $t \times (0.03)$  - chyba

# Útok na šifru KASUMI POPIŠ ÚTOKU

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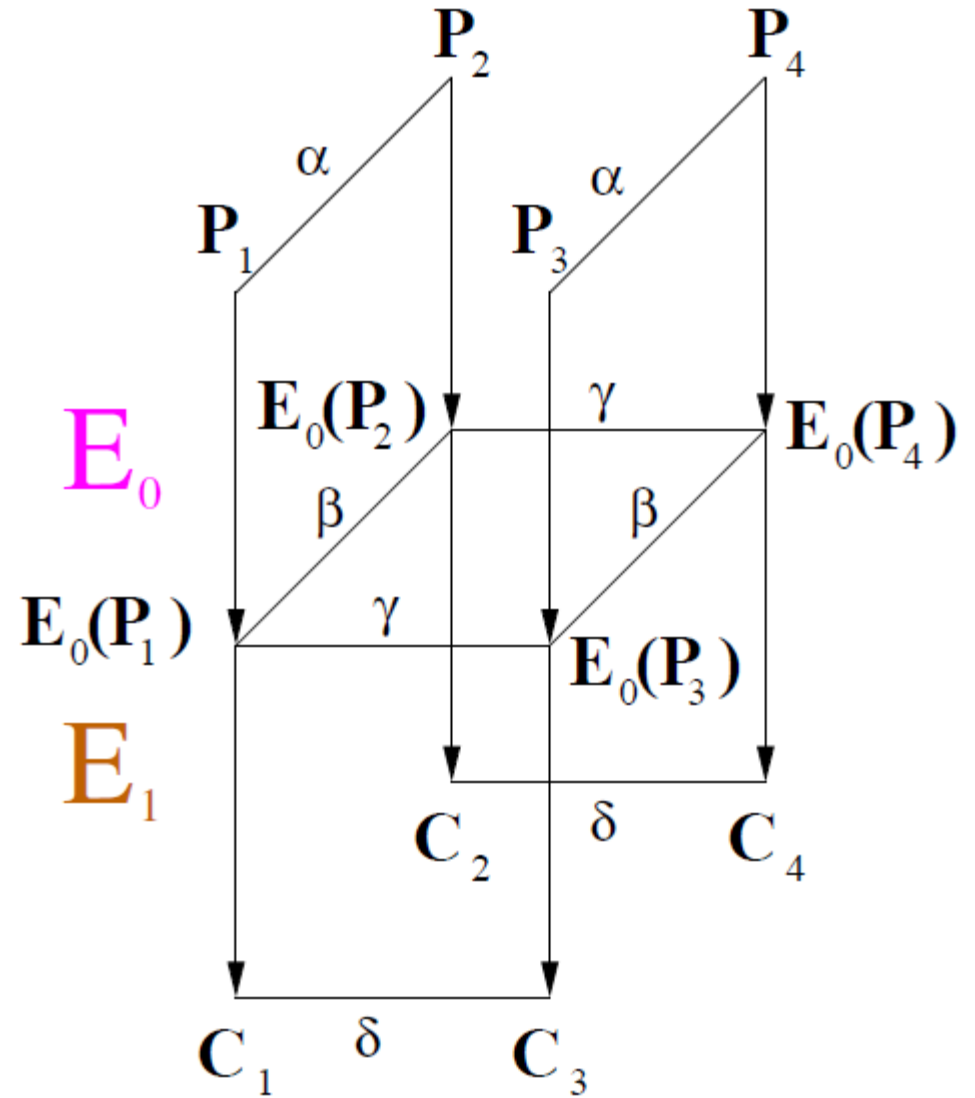
# Boomerang Attack

- $p^2q^2$
- Dobré krátke diferencie
- Zlé dlhé diferencie



# Amplified Boomerang Attack

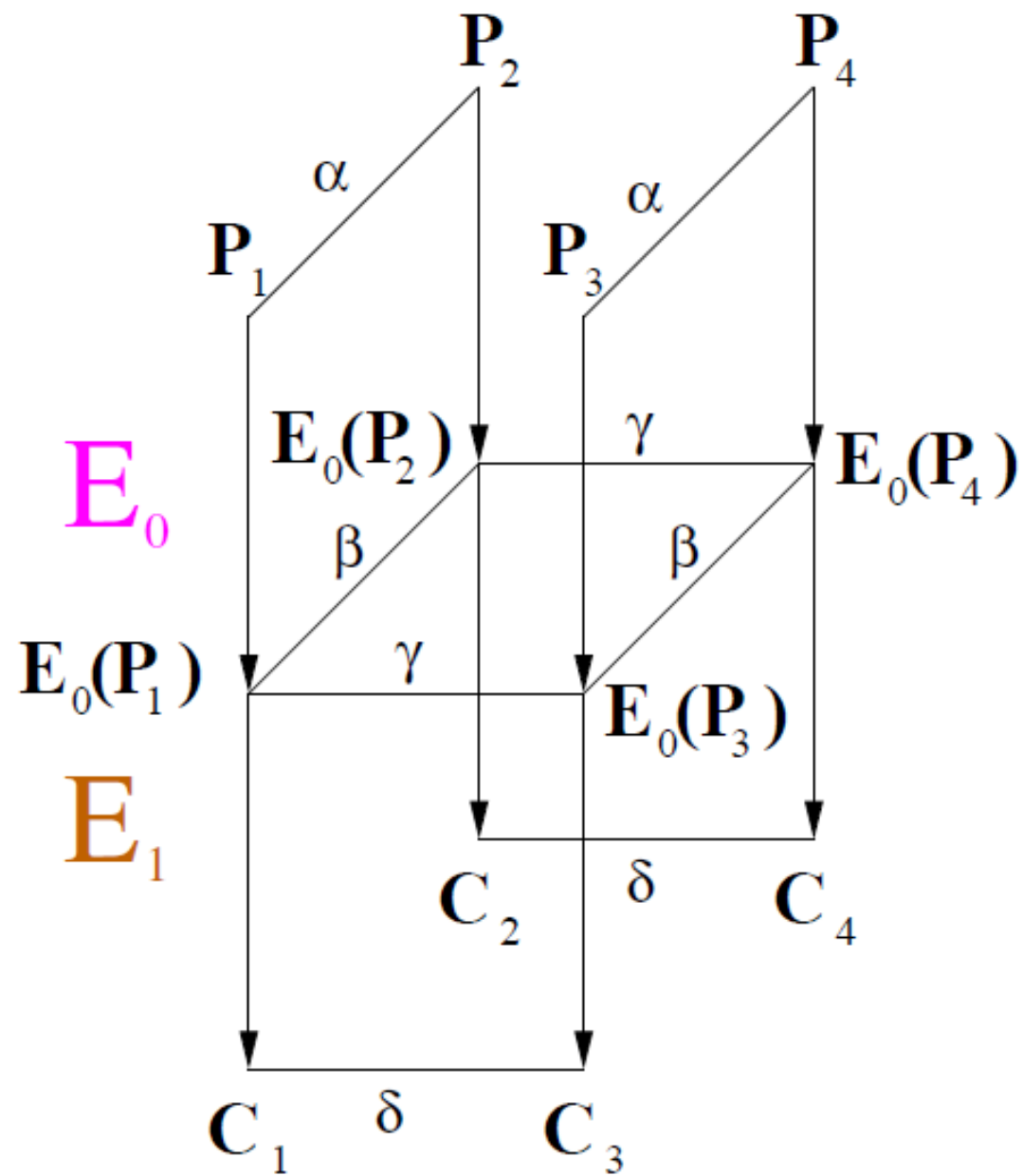
- Zašifruj veľa textu a dúfaj v to, že nejaký bude spĺňať podmienku BA
- $2^{-n-1}p^2q^2$  - pravd. správneho páru
- $2^{-n-1}p^2q^2N^2$
- Aspoň  $2^{\frac{n}{2}+1}$  plaintextov

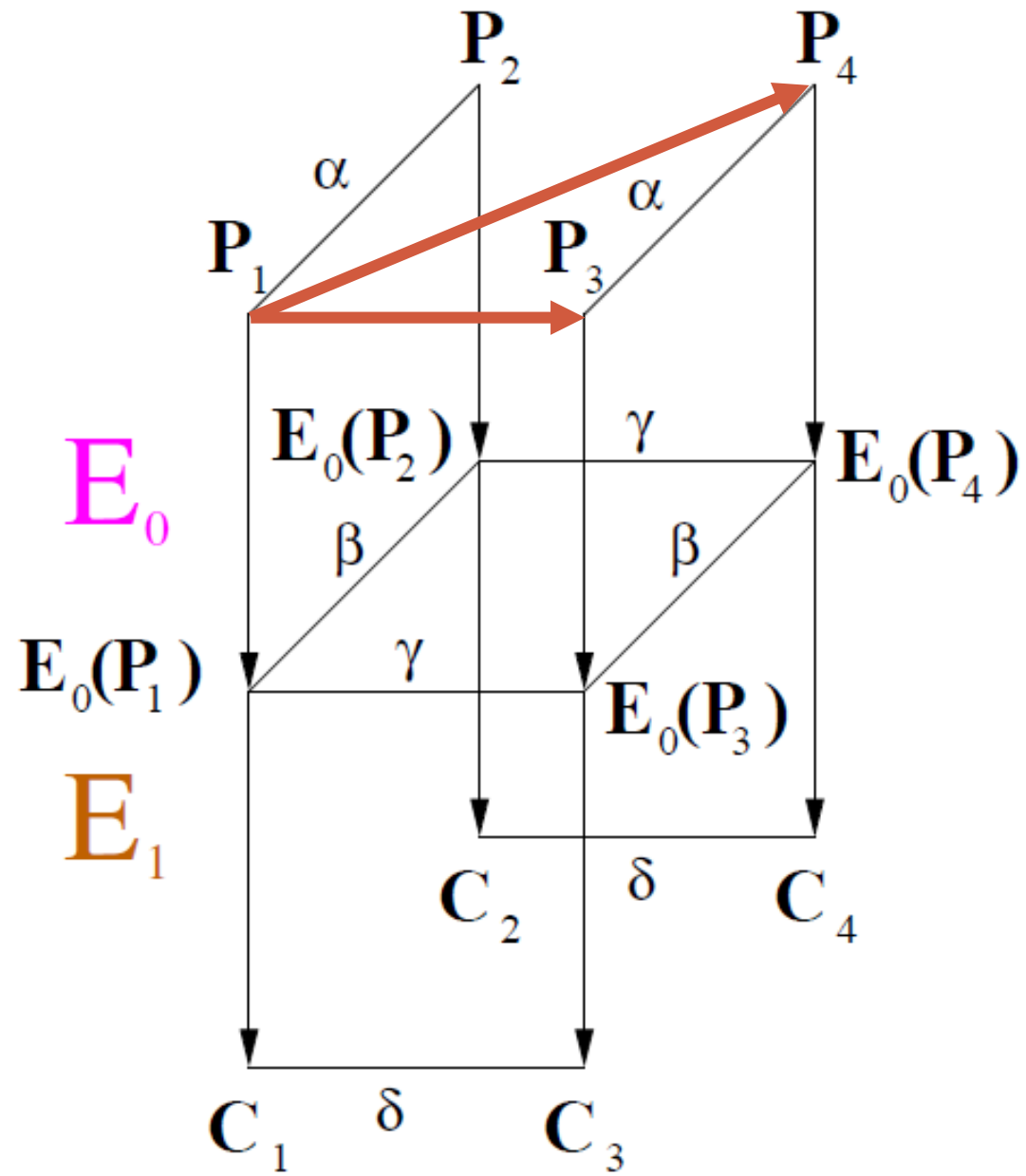




# Rectangle Attack

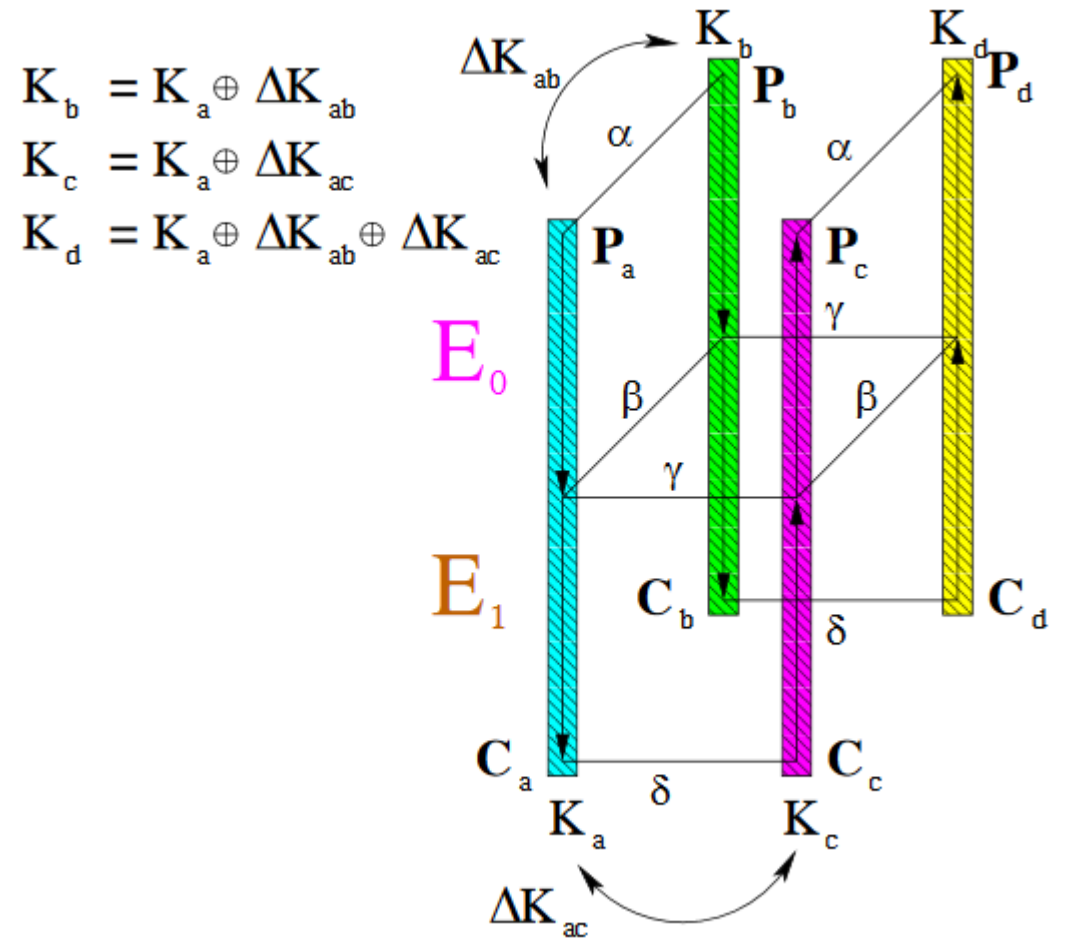
- Rozšířený Amplified Boomerang Attack – len analýzou  $(\gamma \rightarrow \delta) \text{ v } E_1$ ;
  - Štvorica  $((P_1, P_2), (P_3, P_4))$  a k tomu príslušný šifrovaný text  $((C_1, C_2), (C_3, C_4))$  také, že  $P_1 \oplus P_2 = P_3 \oplus P_4 = \alpha$  a  $C_1 \oplus C_3 = C_2 \oplus C_4 = \delta$ , kde  $\alpha$  je vstupná diferenciacia do  $E_0$  a  $\delta$  je výstupná diferenciacia z  $E_1$
  - Využijeme všetky možné  $\gamma$  keď platí  $Z_1 \oplus Z_3 = Z_2 \oplus Z_4 = \gamma$  a  $Z_1 \oplus Z_2 = \beta$ , kde  $Z_i = E_0(P_i)$  ( $\gamma \rightarrow \beta$ ) v  $E_1$
  - Využijeme všetky možné  $\beta$  keď platí  $Z_1 \oplus Z_2 = Z_3 \oplus Z_4$  a  $Z_1 \oplus Z_3 = \gamma$
  - „Pre každý pár je k dispozícii viac párov“  
 $((P_1, P_2), (P_4, P_3))$  a  $((P_1, P_2), (P_3, P_4))$
-





# Related-Key Boomerang Attack

- Vyber náhodne  $P_a$ ,  
a spočítaj  $P_b = P_a \oplus \alpha$
- Požiadaj o šifrovanie  $C_a = E_{K_a}(P_a)$   
a  $C_b = E_{K_b}(P_b)$
- Spočítaj  $C_c = C_a \oplus \delta$  a  $C_d = C_b \oplus \delta$
- Požiadaj dešifrovať  $P_c = E_{K_c}^{-1}(C_c)$   
a  $P_d = E_{K_d}^{-1}(C_d)$
- Skontroluj, či  $P_c \oplus P_d = \alpha$



# Related-Key Rectangle Attack

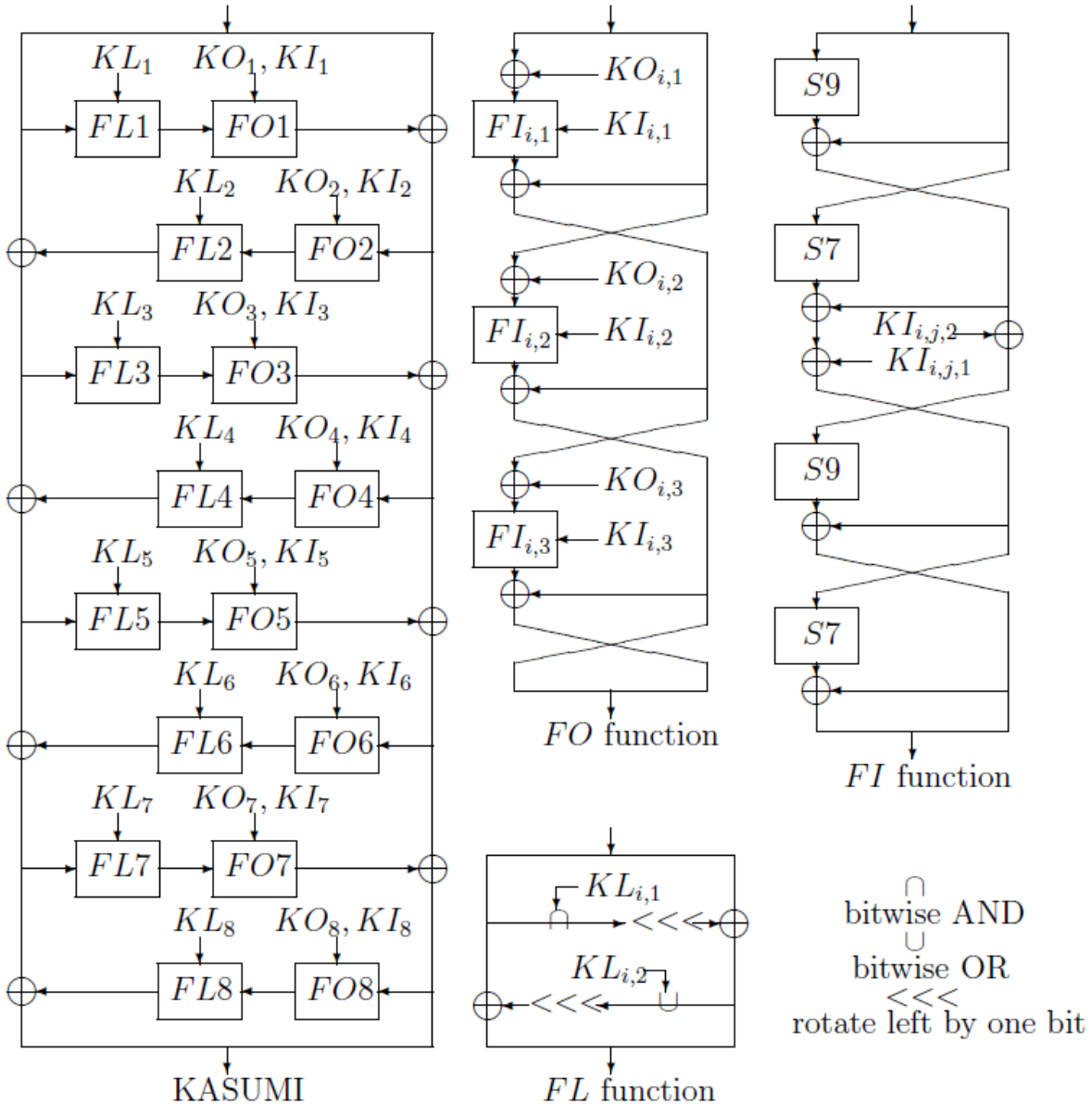
- $K_a, K_b = K_a \oplus K_{ab}, K_c = K_a \oplus K_{ac}, K_d = K_a \oplus K_{ad}$
  - Vyber N otvorených párov  $(P_a, P_b = P_a \oplus \alpha)$  a požiadaj zašifrovanie  $P_a$  kľúčom  $K_a$  a  $P_b$  kľúčom  $K_b$
  - Vyber N otvorených párov  $(P_c, P_d = P_c \oplus \alpha)$  a požiadaj zašifrovanie  $P_c$  kľúčom  $K_c$  a  $P_d$  kľúčom  $K_d$
  - Najdi štvorce  $(P_a, P_b, P_c, P_d)$  a odpovedajúce  $(C_a, C_b, C_c, C_d)$  splňujúce  $C_a \oplus C_c = C_b \oplus C_d = \delta$
  - Lepšie pravdepodobnosti útokov
  - $N^2 2^{-n} (pq)^2$  - správnych štvorcí
-

# Útok na šifru KASUMI sekcia:

Related-Key Boomerang and Rectangle Attacks  
on the Full KASUMI

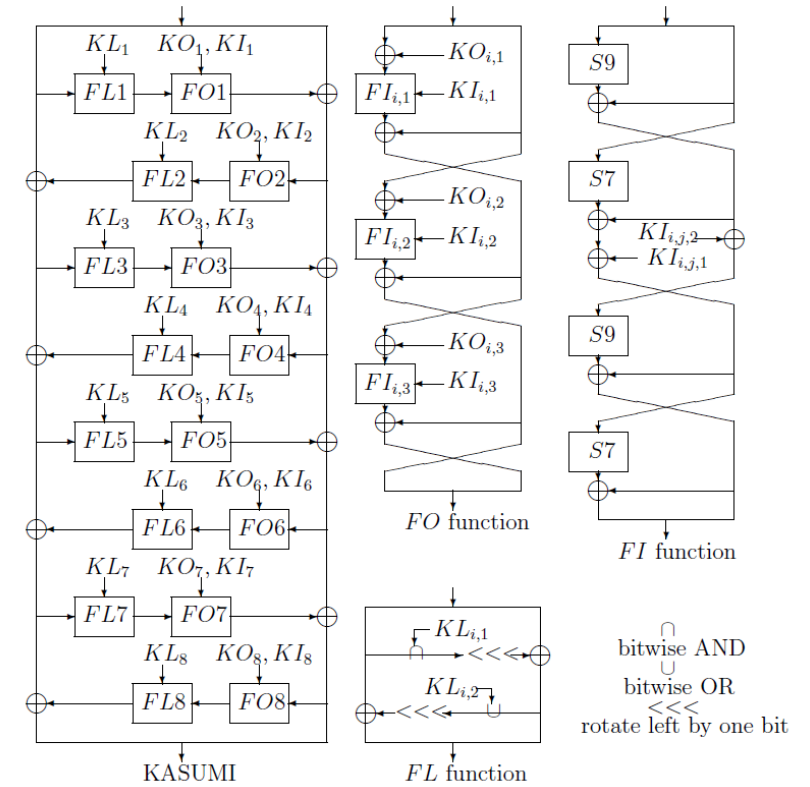
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# KASUMI



# KASUMI 1-4

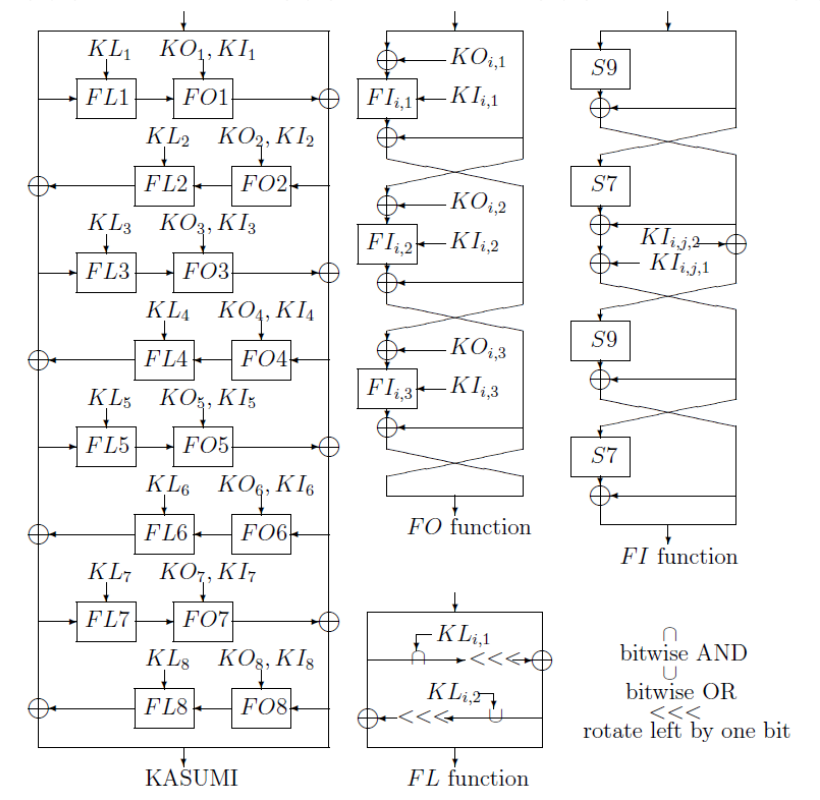
- $\Delta K_{ab} = (0,0,1,0,0,0,0,0)$
- Vstupná diferencia  $\alpha = (0_X, (0020\ 0000)_X)$
- $\alpha = (0_X, (0020\ 0000)_X) \rightarrow (y_X, (0020\ 0000)_X)$
- $2^{-34} = \frac{1}{4} \frac{1}{2^{32}}$
- $P = (P_{LL}P_{LR}P_{RL}P_{RR})$
- $P_{LL}^0 = 0, P_{LR}^1 = 1$  – zlepšime charakteristiku
- $\frac{1}{2^{33}}$ , efektívna pravdepodobnosť  $\frac{1}{2^{17}}$





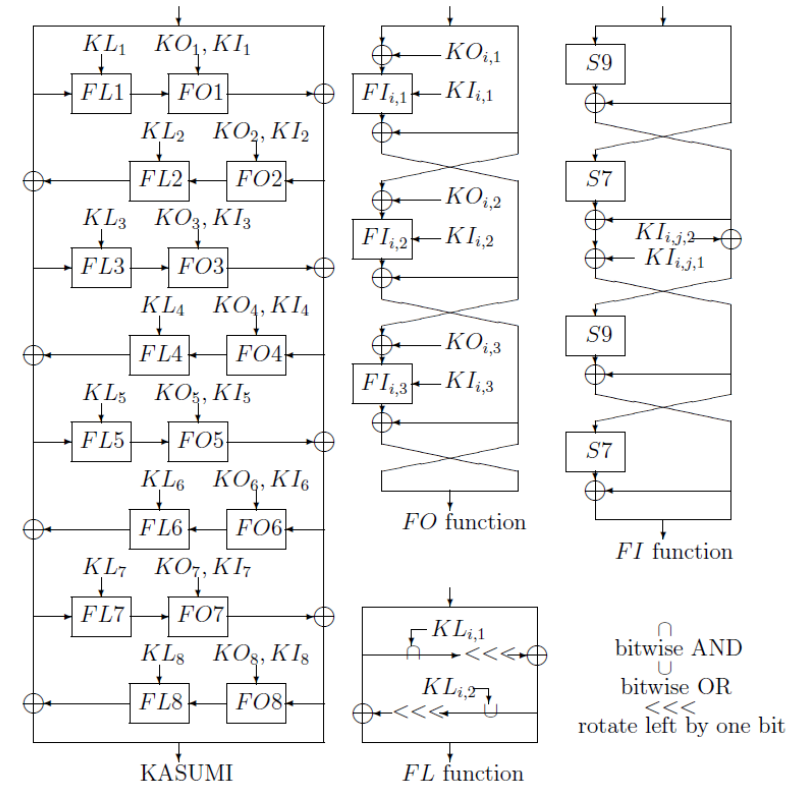
# KASUMI 5-7

- $\Delta K_{ac} = (0,0,0,0,0,0,1,0)$
- Vstupná diferencia  $\gamma = (0_X, (0020\ 0000)_X)$
- $\gamma = (0_X, (0020\ 0000)_X) \rightarrow (0_X, (0020\ 0000)_X)$
- $\frac{1}{4}$ , efektívna je  $\frac{1}{4}$



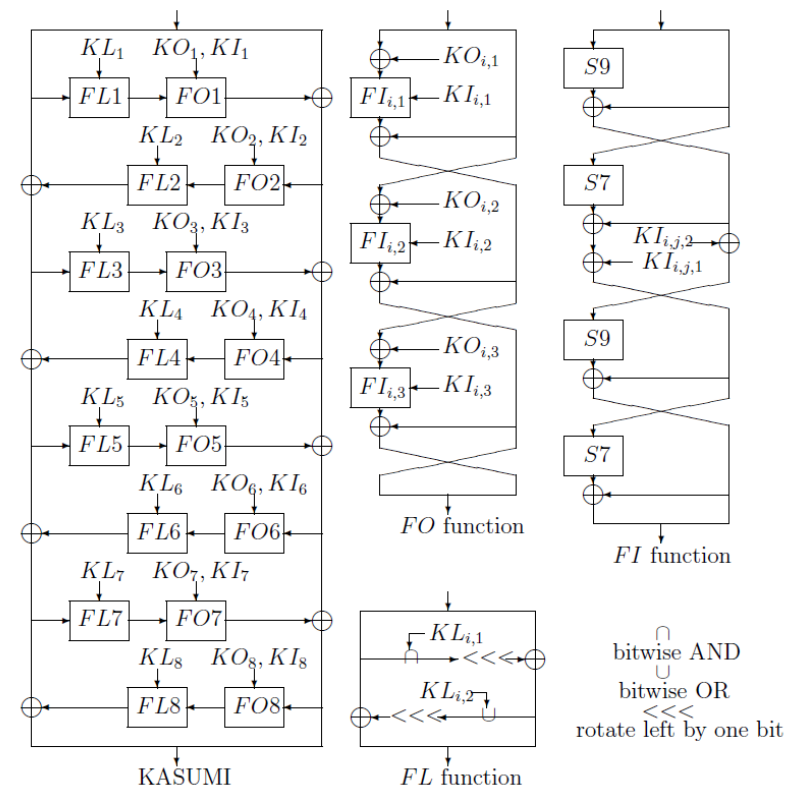
# KASUMI ÚTOK

- $K_a, K_b = K_a \oplus K_{ab}, K_c = K_a \oplus K_{ac}, K_d = K_a \oplus K_{ad}$
- Potrebujeme  $2^{51}$  - otvorených textov
- Celkovo štvoríc máme  $2^{102}$ , potom dostávame 1 správny „obdĺžnik“
- Algoritmus:
  - $2^{51}$  šifrovaní:  $(P_a, P_b = P_a \oplus \alpha), P_{aLL}^0 = 0, P_{aLR}^1 = 1, E(P_a, K_a), E(P_b, K_b)$   
Index:  $(C_{aRL} C_{aRR} C_{bRL} C_{bRR})$
  - $2^{51}$  šifrovaní:  $(P_c, P_d = P_c \oplus \alpha), P_{cLL}^0 = 0, P_{cLR}^1 = 1, E(P_c, K_c), E(P_d, K_d)$   
Index:  $(C_{cRL} \oplus 0020_X, C_{cRR} C_{dRL} \oplus 0020_X, C_{dRR})$  (zhoduje sa diferencia?)



# KASUMI ÚTOK

- $(C_{CRL} \oplus 0020_X, C_{CRR} C_{dRL} \oplus 0020_X, C_{dRR})$ 
  - Nájdeme  $(P_a, P_b)$ , pokračujeme štvoricou  $(P_a, P_b, P_c, P_d)$
- $2^{38}$  približne bude vyhovovať
- Tipneme si kľúč  $(KO_{8,1}, KI_{8,1})$  a vydedukujeme  $KL_{8,2}$  - je možné vypočítať vstupné a výstupné diferencie do OR funkcie (spor)
- Tipneme si kľúč  $(KO_{8,3}, KI_{8,3})$  a vydedukujeme  $KL_{8,2}$  - spočítame diferencie vstupu a výstupu
- Pre všetky vyhovujúce kombinácie urob šifrovanie a over výsledok



# KASUMI ÚTOK – ANALÝZA

- Ohodadovanie  $KL_{8,2}$  –  $2^{-16}$  - zlých kľúčov

OR —  $KL_{8,2}$

$(X'_1, Y'_1)$	$(X'_2, Y'_2)$			
	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	{0,1}	—	1	0
(0,1)	—	—	—	—
(1,0)	1	—	1	—
(1,1)	0	—	—	0

AND —  $KL_{8,1}$

$(X'_1, Y'_1)$	$(X'_2, Y'_2)$			
	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	{0,1}	—	0	1
(0,1)	—	—	—	—
(1,0)	0	—	0	—
(1,1)	1	—	—	1

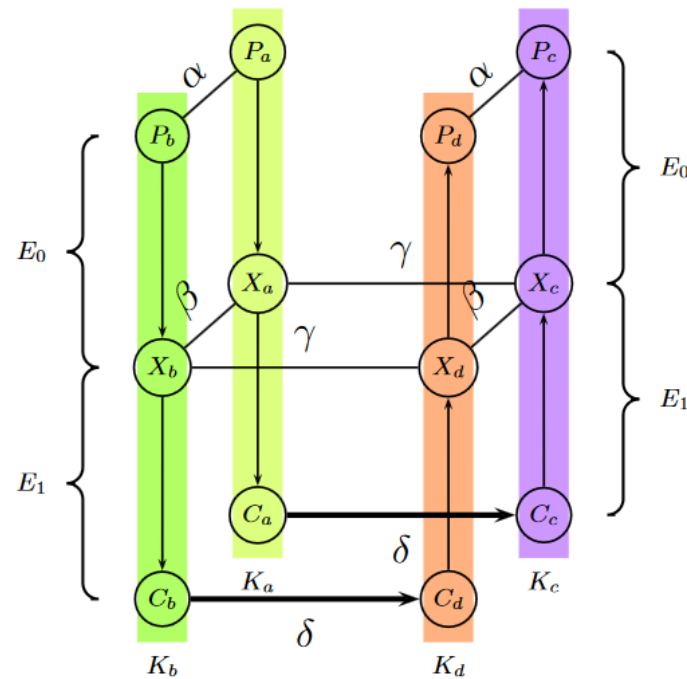
\* The two bits of the differences are denoted by (input difference, output difference):  $(X'_1, Y'_1)$  for one pair and  $(X'_2, Y'_2)$  for the other.

# KASUMI ÚTOK – ANALÝZA

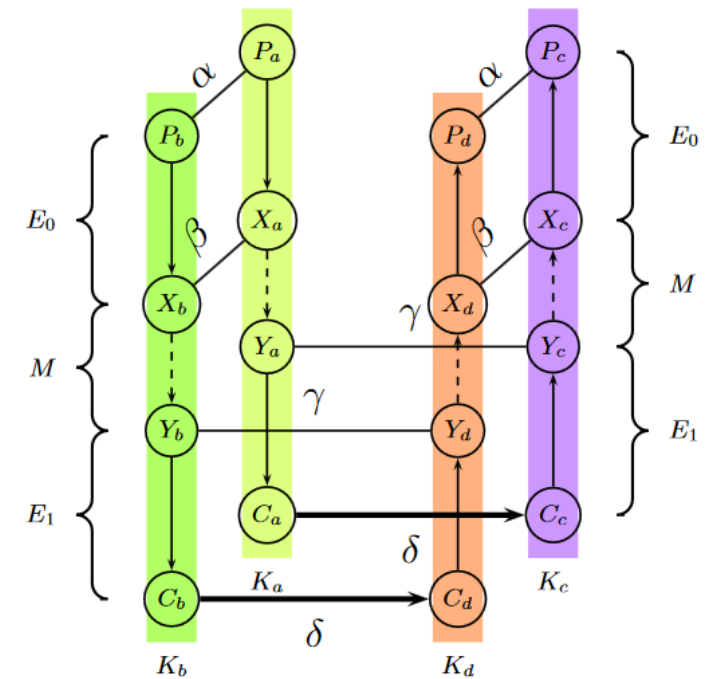
- Ohodadovanie  $KL_{8,2}$  –  $2^{-16}$  - zlých kľúčov
  - $2^{38}2^{32} = 2^{70}$ ;  $2^{70}2^{-16} = 2^{54}$
  - $2^{54}2^{32} = 2^{86}$ ;  $2^{86}2^{-16} = 2^{70}$  - pre 96 bitový kľúč ☺
  - Dokončiť zvyšných 32 bitov
    - $2^{102}$  operácii pri 128 bitovom kľúči
  - Vylepšenia:
    - Pri šifrovaní  $2^{52,6}$  vstupných textov =  $2^{86,6}$
    - Analýzov vieme dostať  $2^{76,1}$  - viac informácií v mojej práci ☺
-

# KASUMI ÚTOK 2

- Sandwich attack
- $2^{32}$  časová zložitost'
- $2^{30}$  paměťová zložitost'
- $2^{25}$  šifrových textov



A Related-Key Boomerang Quartet



A Related-Key Sandwich Quartet

# Zdroj

- E. Biham, O, Dunkelman: Techniques for Cryptanalysis of Block Ciphers (Information Security and Cryptography), Springer, 2017, ISBN 978-3642172311
  - DUNKELMAN, Orr; KELLER, Nathan; SHAMIR, Adi. A practical-time related-key attack on the KASUMI cryptosystem used in GSM and 3G telephony. In: *Annual Cryptology Conference*. Springer Berlin Heidelberg, 2010. p. 393-410.
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**Ďakujem za pozornosť.**

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